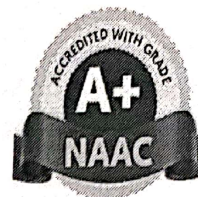


SHRI GNANAMBICA DEGREE COLLEGE – MADANAPALLI

(AUTONOMOUS)
MATHEMATICS - MINOR
SECOND YEAR-III SEMESTER
(W.E.F. Academic Year 2025-26)
COURSE 2: GROUP THEORY



Theory

Credits: 4

5hrs./week

Course Outcomes:

After successful completion of this course, the student will be able to:

1. Demonstrate a clear understanding of the basic definitions, structure, and properties of groups, including examples from various algebraic systems.
2. Explain the concept and importance of subgroups and cosets, and apply methods to identify and analyze them within a group.
3. Understand and verify the criteria for a normal subgroup, and examine its role in group structure and classification.
4. Explore group homomorphisms and isomorphisms, and apply fundamental theorems to solve problems involving group mappings.
5. Analyze the structure and properties of permutation groups and cyclic groups, including the use of cycle notation and identification of generators.

UNIT-I (Groups)

Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – Order of Group – examples – order of an element of group - Composition tables with examples.

UNIT-II (Subgroups, Cosets and Lagrange's Theorem)

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition- examples-criterion for a complex to be a subgroup; Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Coset Definition – properties of Cosets – Index of a subgroups of a finite group – Lagrange's Theorem.

UNIT-III (Normal Subgroups)

Normal Subgroups: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group- Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups - Sub group of index 2 is a normal sub group, Quotient groups.

UNIT-IV (Homomorphism and Isomorphism of Groups)

Definition of homomorphism – Image of homomorphism - Elementary properties of homomorphism – Isomorphism – Automorphism definitions and elementary properties–kernel of a homomorphism – Fundamental theorem on Homomorphism and applications.

UNIT-V (Permutation Groups and Cyclic Groups)

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Cyclic Groups - Definition of cyclic group – elementary properties – classification of cyclic groups.

Activities:

Seminar/ Quiz/ Assignments/ Applications of Group Theory to Real life Problem /Problem Solving Sessions.

Text Book:

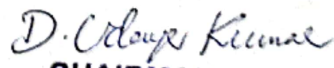
A Textbook of B.Sc. Mathematics in Abstract Algebra, S. Chand Publications.

Reference Books:

1. Modern Algebra by A.R.Vasishtha and A.K.Vasishtha, Krishna Prakashan Media Pvt. Ltd., Meerut.
2. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
3. Modern Algebra by M.L. Khanna, Jai Prakash and Co. Printing Press, Meerut
4. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan


M. R. S.


S. J.


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SHRI GNANAMBICA DEGREE COLLEGE – MADANAPALLI
(AUTONOMOUS)

MATHEMATICS - MINOR
SECOND YEAR-III SEMESTER
COURSE 2: GROUP THEORY

(W.E.F.2025-2026)

Blue Print

TIME: 3Hrs.

MAX. MARKS: 70

SECTION-A

Answer any FIVE of the following questions.

5×4=20

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.
- 10.

SECTION-B

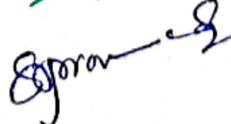
Answer any FIVE of the following questions.

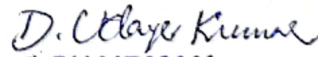
5×10=50

11. (a) Or (b)
12. (a) Or (b)
13. (a) Or (b)
14. (a) Or (b)
15. (a) Or (b)








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SHRI GNANAMBICA DEGREE COLLEGE: MADANAPALLE

(AUTONOMOUS)

MATHEMATICS - MINOR

SEMESTER III-MINOR

COURSE 2: GROUP THEORY

(W.E.F.2025-2026)

Model Question Paper

TIME: 3Hrs.

MAX. MARKS: 70

SECTION-A

Answer any **FIVE** of the following questions.

$5 \times 4 = 20$

- 1) Prove that the set of fourth roots of unity forms an Abelian group w.r.t. multiplication.
- 2) Prove that the set $G = \{a + b\sqrt{2} / a, b \in \mathbb{Q}\}$ is a commutative group w.r.t. addition.
- 3) H is any subgroup of a group G . Prove that $HH = H$.
- 4) Prove that the intersection of two subgroups is also a subgroup.
- 5) Prove that any two left or right cosets of a subgroup are either disjoint or identical.
- 6) G is a group and H is a subgroup of index 2 in G . Prove that H is normal subgroup of G .
- 7) Prove that every homomorphic image of a commutative group is commutative.
- 8) If for a group G , $f: G \rightarrow G$ is given by $f(x) = x^2, x \in G$ is a homomorphism, prove that G is abelian.
- 9) Find the order of the cycle $(1 \quad 4 \quad 5 \quad 7)$
- 10) Prove that a group of prime order is cyclic.

SECTION-B

Answer **ALL** of the following questions.

$5 \times 10 = 50$

- 11) (a) Prove that a finite semi group $(G, .)$ satisfying the cancellation laws is a group.

Or

(b) Prove that the set $\mathbb{Z}$ of all integers is a group with respect to the operation $*$ defined by $a*b = a+b+2$ for all a, b of $\mathbb{Z}$.

- 12) (a) Prove that the necessary and sufficient condition for a complex H of a finite group G to be a subgroup of G is $a, b \in H \Rightarrow ab \in H$.

Or

(b) Prove that the union of two subgroups of a group is a subgroup if and only if one is contained in the other.

13) (a) State and prove Lagrange's theorem for finite groups.

Or

(b) Prove that a subgroup H of a group G is normal iff every left coset of H is a right coset of H in G .

14) (a) Prove that the necessary and sufficient condition for a homomorphism ' f ' of group G onto a group G' with kernel K to be an isomorphism of G into G' is that $K = \{e\}$.

Or

(b) State and prove fundamental theorem of homomorphism on groups.

15) (a) State and prove Cayley's theorem.

Or

(b) Prove that every subgroup of cyclic group is cyclic.


M. D. Sesh


D. Velayutham

D. Velayutham
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